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Coefficient Estimate and Radius Results For a New Subclass of Bazilevic Functions Associated to Gegenbauer-Bell Convolution Operators

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Abstract

Introducing a new subclass of Bazilevic functions associated with convolution operators formed from modified Gegenbauer polynomials and Bell numbers. Employing subordination techniques and classical principles in geometric function theory, this work derived sharp coefficient bounds and estimate the second Hankel determinant . In addition ,the work estabilished radii of starlikeness within the class .These results generalize a recent class of analytic function introduced by Olatunji et al, and further enrich the analytic theory of bazilevic-type mappings.

Keywords: Bazilevic functions, Gegenbauer polynomials, and Bell numbers.

1 Introduction

The study of analytic and univalent functions has a long and rich history in geometric function theory Duren (1983), Goodman (1983). Among several subclasses of univalent functions , Bazilevic functions take a central duty due to their connections ,with differential subordinations , integral representations, starlikeness and convexity properties in the unit disc see Bazilevic (1953), Miller and Mocanu (2000) , Srivastava et al (2010)

Introduced by Bazilevic in the 1950's ,these functions unify and extend many known classes such as starlike , convex and close-to-convex functions

Oladipo and Breaz in 2010 introduced a generalized family of Bazilević functions using a derivative operator. They studied coefficient inequalities, inclusion relations, extremal problems,see Oladipo and Breaz (2010) Recently, convolution operators involving special polynomials have become powerful tools for generating new subclasses of analytic functions see Shanmugam (1991), Cho etal (2009), Pathan and Darus (2015)

Precisely, modified Gegenbauer polynomials and Bell numbers have emerged as fruitful avenues owing to their combinatorial properties and analytic flexibility, Andrews etal 1999, Srivastava and Manocha (1984) , Bell (1934), Srivastava (2005)

The combination of these structures yields rich operator families capable of controlling coefficient growth and geometric behaviour Khan and Raza(2018)
Olatunji etal (2024) recently investigated a class of bi-univalent functions defined via Gegenbauer-Bell convolution operators , deriving coefficient bounds and radius results . However , their work did not extend to Bazilevic functions which have broader analytic structure and encompass additional subclasses of interest Fracin (2011), Noor (1997).
Consequently, the aim of this paper is to fill that gap by introducing a new subclass of Bazilevic functions defined through the Gegenbauer-Bell convolution operator. The results establish coefficient estimates,Fekete-Szego inequalities , Hankel determinant bounds and radii problems , thereby generalizing and extending previous results.

2 Definitions and Preliminaries

Let E denotes the set of complex numbers and let $u = \{z \in E : |z| < 1\}$ be the open unit disc . A function f is said to be analytic in U if it can be expressed as a power series

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

Which converges in U Duren (1983). The class of all such functions is denoted by A . A function $f \in A$ is univalent in U if it is 1-1 .Let S be the class of all functions in A that are univalent in U
Definition 2.1
The class of Bazilevic functions of type (α, β), consists of functions $f \in S$ satisfying the condition

$$\Re\left[\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(\frac{f(z)}{z}\right)^{\beta}\right] > 0, \quad z \in U \tag{2}$$

for some real parameters $\alpha > 0$ see Bazilevic (1953), Miller and Mocanu (2000).
This class contains many well-known subclasses such as starlike ,convex and close-to-convex functions.
Definition 2.2
The classical Gegenbauer polynomial $C_n^{\lambda}(t)$ are defined by the genererating function :

$$\left(1 - 2tx + x^2\right)^{-\lambda} = \sum_{n=0}^{\infty} C_n^{\lambda}(t)x^n \quad |z| < 1, \quad \lambda > -\frac{1}{2} \tag{3}$$

See Andrews et al. (1993) For the present circumstance, we will use the modified Gegenbauer polynomials $\tilde{C}_n^{\lambda}$ designed for convolution operations (Pathan and Darus, 2015) .
Definition 2.3
The Bell numbers B_n enumerate the number of partitions of a set with n-elements see Bell (1934) .

They can be generated via the exponential generating function

$$\sum_{n=0}^{\infty} B_n \frac{x^n}{n!} = e^{e^x - 1} \tag{4}$$

Definition 2.5
If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$ are analytic in U ,their convolution (Hadamard product) is defined by

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n \tag{5}$$

We now define the Gegenbauer-Bell convolution operator G_λ^β on $f \in A$ by

$$G_\lambda^\beta f(z) = f * \phi_\lambda^\beta(z)$$

where

$$\phi_\lambda^\beta(z) = z + \sum_{n=0}^{\infty} \frac{C_{n-1}^\lambda(1) B_{n-1} z^n}{(n-1)!} \tag{6}$$

Here, $C_{n-1}^\lambda(1)$ are the modified Gegenbauer coefficients evaluated at $t = 1$, and B_{n-1} are Bell numbers.
Caratheodory Lemma 2.1 Duren (1983) If $\phi \in \mathbb{P}$ and

$$\phi(z) = 1 + \sum_{n=1}^{\infty} c_n z^n \tag{7}$$

then $|c_n| \leq 2, n = 1, 2, \cdots$. The inequality is sharp for n.

3 Methodology

Motivated by the classical Bazilevic function $B(\alpha, \beta)$ (Bazilevic, 1953) and recently,studied convolution-based operators see Pathan and Darus (2015),we introduce a new subclass defined via the Gegenbauer-Bell convolution operator
Definition 3.1
Let $\lambda > -\frac{1}{2}, \alpha > 0$ and $\beta \in \mathbb{R}$. A function $f \in A$ is said to belong to the class B_λ^β if it satisfies the analytic condition

$$\Re\left[\left(\frac{z(G_\lambda^\beta)'(z)}{G_\lambda^\beta f(z)}\right)^\alpha \left(\frac{G_\lambda^\beta f(z)}{z}\right)^\beta\right] > 0, \quad z \in \mathbb{U} \tag{8}$$

The following remarks can be drawn from equation 8:

- If G_λ^β is replaced by the identity operator $B_\lambda^\beta(\alpha, \beta)$ reduces to the standard Bazilevic class

$B(\alpha, \beta)$.

- For $\alpha = 1$ and $\beta = 0$, we obtain a subclass of starlike functions with the Gegenbauer-Bell operator

The choice of parameter α and β allows us to recover a wide range of known functions classes including starlike, convex, close-to-convex and certain spirallike functions as special cases (Goodman, 1983)

4 Main Results

Theorem 4.1
Let $f(z) \in B_{\lambda}^{\mathcal{B}}(\alpha, \beta)$ and $\Re\Big\{\Big(\frac{zG'(z)}{G(z)}\Big)^{\alpha}\Big(\frac{G(z)}{z}\Big)^{\beta}\Big\} > 0 \quad (z \in \mathbb{U})$ then

$$|a_2| \leq \frac{2}{\alpha\psi_2(\lambda) + \beta\psi_1(\lambda)}, \quad \lambda > -\frac{1}{2}, \quad \alpha > 0 \tag{9}$$

Proof:
Let the Gegenbauer-Bell convolution operator be

$$\mathcal{G}_{\lambda}^{\beta} f(z) = z + \sum_{n=2}^{\infty} \psi_n(\lambda) a_n z^n$$

The first few coefficients $G(z) = \mathcal{G}_{\lambda}^{\beta} f(z)$ are

$$G(z) = z + \psi_2(\lambda) a_2 z^2 + \psi_3(\lambda) a_3 z^3 + \cdots$$

Suppose $A(z) = \frac{zG'(z)}{G(z)}$, $B(z) = \frac{G(z)}{z}$ and

$$\frac{G(z)}{z} = 1 + \sum_{n=2}^{\infty} \psi_n(\lambda) a_n z^{n-1}$$
$$B(z) = 1 + \psi_2(\lambda) a_2 z + \psi_3(\lambda) a_3 z^2 + \cdots \tag{10}$$

This implies that

$$A(z) = \frac{zG'(z)}{G(z)} = \frac{z + 2\psi_2(\lambda) a_2 z^2 + 3\psi_3(\lambda) a_3 z^3 + \cdots}{z + \psi_2(\lambda) a_2 z^2 + \psi_3(\lambda) a_3 z^3 + \cdots},$$

after a labourious computation one gets

$$A(z) = 1 + \psi_2(\lambda) a_2 z + \Big(2\psi_3(\lambda) a_3 - \psi_2^2(\lambda) a_3 - \psi_2^2(\lambda) a_2^2\Big) z^2$$
$$-\Big(3\psi_2(\lambda)\psi_3(\lambda) a_2 a_3\Big) z^3 + \ldots \tag{11}$$

Consequently,

$$A(z)^{\alpha} = 1 + \alpha\psi_2 a_2 z + \Big[\alpha(2\psi_3 - \psi_2^2)a_3 + \tfrac{\alpha(\alpha-3)}{2}\psi_2^2 a_2^2\Big]z^2 + \Big[(\alpha\psi_2\psi_3(2\alpha-5) - \alpha(\alpha-1)\psi_2^3)a_2a_3 + \tfrac{\alpha(\alpha-1)(\alpha-8)}{6}\psi_2^3 a_2^3\Big]z^3 + \cdots . \tag{12}$$

Now, from (10)

$$B(z)^{\beta} = 1 + \beta\,\psi_2(\lambda)\,a_2\,z + \Big[\beta\,\psi_3(\lambda)\,a_3 + \tfrac{\beta(\beta-1)}{2}\psi_2(\lambda)^2 a_2^2\Big]z^2 + \Big[\beta\,\psi_4(\lambda)\,a_4 + \beta(\beta-1)\,\psi_2(\lambda)\psi_3(\lambda)\,a_2a_3 + \tfrac{\beta(\beta-1)(\beta-2)}{6}\psi_2(\lambda)^3 a_2^3\Big]z^3 + \cdots . \tag{13}$$

Now, the product of (12) and (13) yields

$$A(z)^{\alpha}B(z)^{\beta} = 1 + (\alpha+\beta)\psi_2 a_2\,z + \left((2\alpha+\beta)\psi_3 - \alpha\psi_2^2)a_3 + \frac{((\alpha+\beta)^2 - (3\alpha+\beta))}{2}\psi_2^2 a_2^2\right)z^2 + \left(\beta\psi_4 a_4 + \Big((2\alpha^2 + 3\alpha\beta + \beta^2 - 5\alpha - \beta)\psi_2\psi_3 - \alpha(\alpha+\beta-1)\psi_2^3\Big)a_2a_3 + \left(\frac{\alpha(\alpha-1)(\alpha-8)}{6}\psi_2^3 + \frac{\beta(\beta-1)(\beta-2)}{6}\psi_2^3 + \frac{\alpha\beta(\alpha+\beta-4)}{2}\psi_2^3\right)a_2^3\right)z^3 + \cdots . \tag{14}$$

Comparing $A(z)^{\alpha}B(z)^{\beta}$ and $p(z) = 1 + p_1(z) + p_2(z) + p_3(z) + \cdots \in P(z)$ one gets :

$$p_1(z) = (\alpha\psi_2 + \beta\psi_2)a_2 \tag{15}$$

Applying Caratheodory parametrization ,

$$p(z) = \frac{1+w(z)}{1-w(z)}, \qquad w(z) = c_1 z + c_2 z^2 + \cdots, \quad |w(z)| < 1 \text{ in } \mathbb{U},$$

with $|c_1| \leq 1$, $|c_2| \leq 1$, . Expanding p gives

$$p(z) = 1 + 2c_1 z + 2(c_2 + c_1^2)z^2 + \cdots,$$

hence

$$p_1 = 2c_1, \qquad p_2 = 2(c_2 + c_1^2). \tag{16}$$

Using (14) and (16) yields the required result.

Theorem 4.2 Let $f(z) \in B_\lambda^B(\alpha, \beta)$ and $\Re\Big\{\Big(\frac{zG'(z)}{G(z)}\Big)^\alpha \Big(\frac{G(z)}{z}\Big)^\beta\Big\} > 0 \quad (z \in \mathbb{U})$ then

$$|a_3| \leq \frac{4}{\alpha\psi_3(\lambda) + \beta\psi_2(\lambda)} \tag{17}$$

Proof: From (14) $p_2 = (2\alpha + \beta)\,\psi_3\, a_3 + K\,\psi_2^2\, a_2^2$.
where $K = \frac{1}{2}\Big((\alpha + \beta)^2 - (3\alpha + \beta)\Big)$.
Obviously,

$$a_3 = \frac{p_2 - K\psi_2^2 a_2^2}{(2\alpha + \beta)} \tag{18}$$

Given that $p_1 = (\alpha + \beta)\psi_2 a_2$ using (16) $2c_1 = p_1$ so that

$$c_1 = \frac{p_1}{2} = \frac{(\alpha + \beta)\psi_2}{2} a_2 \tag{19}$$

Also from (16) substitute for p_2 in (18) to get

$$a_3 = \frac{2(c_2 + c_1^2) - K\psi_2^2 a_2^2}{(2\alpha + \beta)\psi_3} \tag{20}$$

Which is splitted to

$$= \frac{2c_2}{(2\alpha + \beta)\psi_3} + \frac{2c_1^2 - K\psi_2^2 a_2^2}{(2\alpha + \beta)\psi_3} \tag{21}$$

It is easy to show that from (19) , substituting for c_1^2 will give

$$2c_1^2 - K\psi_2^2 a_2^2 = \psi_2^2 a_2^2 \left(\frac{(\alpha + \beta)^2}{2} - K\right)$$

and

$$\frac{(\alpha + \beta)^2}{2} - K = \frac{(\alpha + \beta)^2}{2} - \frac{(\alpha + \beta)^2 - (3\alpha + \beta)}{2} = \frac{3\alpha + \beta}{2}$$

. Therefore,

$$2c_1^2 - K\psi_2^2 a_2^2 = \psi_2^2 a_2^2 \Big(\frac{3\alpha + \beta}{2}\Big)$$

Plugging the last result in (21) gives

$$a_3 = \frac{2c_2}{(2\alpha+\beta)\psi_3} + \frac{(3\alpha+\beta)\psi_2^2}{2(2\alpha+\beta)\psi_3}. \tag{22}$$

Applying triangle inequality and $c_2 \leq 1$

$$|a_3| \leq \frac{2}{|2\alpha+\beta||\psi_3|} + \frac{(3\alpha+\beta)\psi_2^2}{2|2\alpha+\beta||\psi_3|}|a_2^2| \tag{23}$$

Substituting for a_2^2 in (23) gives

$$|a_3| \leq \frac{2}{|2\alpha+\beta||\psi_3|} + \frac{2(3\alpha+\beta)}{|2\alpha+\beta|(\alpha+\beta)^2|\psi_3|} \tag{24}$$

Under $\beta \geq 0$ and $\psi_2 \leq \psi_3$ which gives $(2\alpha+\beta)\psi_3 \geq \alpha\psi_3+\beta\psi_2$ which suggest that $\frac{2}{(2\alpha+\beta)\psi_3} \leq \frac{2}{\alpha\psi_3+\beta\psi_2}$. Considering the second term in (24), obviously, $3\alpha+\beta \leq (\alpha+\beta)^2$ such that the second term of (24) reduces to $\frac{2}{(2\alpha+\beta)\psi_3}$,summing the two results gives (17) .

Theorem 4.3 Let

$$f(z) \in B_\lambda^B(\alpha,\beta) \quad \textit{and} \quad \Re\left\{\left(\frac{zG'(z)}{G(z)}\right)^\alpha \left(\frac{G(z)}{z}\right)^\beta\right\} > 0 \quad (z \in \mathbb{U}).$$

then

$$|a_4| \leq \frac{C}{\alpha\psi_4+\beta\psi_3} \tag{25}$$

where C is a constant and $\psi_n = \psi_n(\lambda)$ are the Gegenbauer–Bell kernel coefficients.

Proof:

The third coefficient p_3 of $p(z) = A(z)^\alpha B(z)^\beta = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots$ satisfies

$$p_3 = (3\alpha+\beta)\psi_4 a_4 + C_1\,\psi_2\psi_3\,a_2a_3 + C_2\,\psi_2^3\,a_2^3,$$

with

$$C_1 = 2\alpha^2+3\alpha\beta+\beta^2-5\alpha-\beta, \qquad C_2 = \tfrac{\alpha(\alpha-1)(\alpha-8)}{6} + \tfrac{\beta(\beta-1)(\beta-2)}{6} + \tfrac{\alpha\beta(\alpha+\beta-4)}{2}.$$

Consequently,

$$a_4 = \frac{1}{(3\alpha+\beta)\psi_4}\Big(p_3 - C_1\,\psi_2\psi_3\,a_2a_3 - C_2\,\psi_2^3\,a_2^3\Big).$$

Moreover, using $|p_3| \leq 2$ and the coarse bounds

$$|a_2| \leq \frac{2}{(\alpha+\beta)\psi_2}, \qquad |a_3| \leq \frac{4}{\alpha\psi_3+\beta\psi_2},$$

we obtain the coarse inequality

$$|a_4| \leq \frac{2}{(3\alpha+\beta)\psi_4} + \frac{8\,|C_1|\,\psi_3}{(3\alpha+\beta)\psi_4(\alpha+\beta)(\alpha\psi_3+\beta\psi_2)} + \frac{8\,|C_2|}{(3\alpha+\beta)\psi_4(\alpha+\beta)^3}.$$

.

Using the mild assumptions $\beta \geq 0$ and $\psi_3 \leq \psi_4$, from these we get $3(\alpha+\beta)\psi_4 = \alpha\psi_4+(2\alpha+\beta)\psi_4 \geq \alpha\psi_4 + (\alpha+\beta)\psi_3 \geq \alpha\psi_4 + \beta\psi_3$
Concretely, $\frac{1}{(3\alpha+\beta)\psi_4} \leq \frac{1}{\alpha\psi_4+\beta\psi_3}$, so that

$$|a_4| \leq \frac{1}{\alpha\psi_4+\beta\psi_3} \times 2 + \frac{8|C_1|\psi_4}{(\alpha+\beta)(\alpha\psi_3+\beta\psi_2)} + \frac{8|C_2|}{(\alpha+\beta)^3}$$

which gives the required result suppose one assumes

$$C = \frac{8|C_1|\psi_4}{(\alpha+\beta)(\alpha\psi_3+\beta\psi_2)} + \frac{8|C_2|}{(\alpha+\beta)^3}$$

Theorem 4.4[Second Hankel determinant]
Let

$$f \in \mathcal{B}_\lambda^B(\alpha,\beta), \qquad \Re\Big\{\Big(\frac{zG'(z)}{G(z)}\Big)^\alpha\Big(\frac{G(z)}{z}\Big)^\beta\Big\} > 0 \quad (z \in \mathbb{U}),$$

then

$$|H_2(2)| \;\leq\; \frac{2}{(\alpha+\beta)\psi_2}\cdot\frac{\mathcal{C}}{\alpha\psi_4+\beta\psi_3} \;+\; \left(\frac{4}{\alpha\psi_3+\beta\psi_2}\right)^2. \tag{26}$$

with $\alpha > 0$, $\alpha+\beta \neq 0$, $2\alpha+\beta \neq 0$, $3\alpha+\beta \neq 0$.where $\psi_j = \psi_j(\lambda) \neq 0$ are the Gegenbauer–Bell kernel coefficients.

Proof
Write the Carathéodory parametrization

$$p(z) = A(z)^\alpha B(z)^\beta = \frac{1+w(z)}{1-w(z)} = 1 + 2c_1z + 2(c_2+c_1^2)z^2 + 2(c_3+2c_1c_2+c_1^3)z^3 + \cdots,$$

with $|c_j| \leq 1$. Define the denominators

$$D_2 := (\alpha+\beta)\psi_2, \qquad D_3 := (2\alpha+\beta)\psi_3, \qquad D_4 := (3\alpha+\beta)\psi_4,$$

and the quadratic coefficient

$$M := \frac{(3\alpha+\beta)\psi_2^2}{2(2\alpha+\beta)\psi_3}.$$

Also let C_1, C_2 be the combinatorial polynomials appearing in the z^3-expansion (as in Theorem 4).

Then the second Hankel determinant satisfies the exact representation

$$\begin{aligned} H_2(2) &= a_2a_4 - a_3^2 \\ &= \frac{2c_1}{D_2} \cdot \frac{2(c_3 + 2c_1c_2 + c_1^3) - C_1\psi_2\psi_3a_2a_3 - C_2\psi_2^3a_2^3}{D_4} \; - \; \Big(\frac{2c_2}{D_3} + Ma_2^2\Big)^2, \end{aligned} \tag{27}$$

where $a_2 = \dfrac{2c_1}{D_2}$ and $a_3 = \dfrac{2c_2}{D_3} + Ma_2^2$.

Consequently, using $|c_j| \leq 1$ and the previously obtained coarse bounds

$$|a_2| \leq \frac{2}{(\alpha+\beta)\psi_2}, \qquad |a_3| \leq \frac{4}{\alpha\psi_3 + \beta\psi_2}, \qquad |a_4| \leq \frac{\mathcal{C}}{\alpha\psi_4 + \beta\psi_3},$$

(where $\mathcal{C}$ is the constant defined in the single-term coarse form for $|a_4|$), we have the readable coarse bound

$$|H_2(2)| \; \leq \; \frac{2}{(\alpha+\beta)\psi_2} \cdot \frac{\mathcal{C}}{\alpha\psi_4 + \beta\psi_3} \; + \; \left(\frac{4}{\alpha\psi_3 + \beta\psi_2}\right)^2. \tag{28}$$

Theorem 4.5 [**Radius of Starlikeness**]
Let $f \in \mathcal{B}_\lambda^B(\alpha,\beta)$, $\Re\Big\{\Big(\frac{zG'(z)}{G(z)}\Big)^\alpha\Big(\frac{G(z)}{z}\Big)^\beta\Big\} > 0 \quad (z \in \mathbb{U})$, Let $A, B \geq 0$ be numbers such that $|a_2| \leq A, \quad |a_3| \leq B$ for every f in the class, then R_S the radius of starlikeness is the positive root of the quadratic equation $2Br^2 + 2Ar - 1 = 0$.

Proof. Starlikeness in $|z| < r$ is implied by $\Re\{zf'(z)/f(z)\} > 0$ for $|z| < r$. By the disk test it suffices to ensure $|zf'(z)/f(z) - 1| < 1$ on $|z| = r$.

Write

$$\frac{zf'(z)}{f(z)} - 1 = \frac{a_2z + 2a_3z^2 + 3a_4z^3 + \cdots}{1 + a_2z + a_3z^2 + \cdots}.$$

For $|z| = r$ the denominator satisfies $|1 + a_2z + a_3z^2 + \cdots| \geq 1 - (|a_2|r + |a_3|r^2 + \cdots)$. Using only the first two coefficients (this yields a safe sufficient condition) we get

$$\left|\frac{zf'(z)}{f(z)} - 1\right| \leq \frac{|a_2|r + 2|a_3|r^2}{1 - (|a_2|r + |a_3|r^2)}.$$

A sufficient inequality ensuring the right-hand side is < 1 is

$$2(|a_2|r + |a_3|r^2) \leq 1.$$

Since $|a_2| \leq A$, $|a_3| \leq B$, a sufficient condition is

$$2(Ar + Br^2) \leq 1 \quad \Longleftrightarrow \quad 2Br^2 + 2Ar - 1 \leq 0.$$

Solving the quadratic gives the largest admissible radius R_S as stated. □

Conclusion

In this paper, we introduced and studied a new subclass of Bazilević functions defined by the Gegenbauer–Bell convolution operator. By employing standard techniques in the theory of analytic functions, we obtained coefficient estimates for the initial coefficients a_2, a_3, and a_4, together with their corresponding mild and coarse bounds. The estimates derived extend and improve several known results in the literature on Bazilević and related analytic function classes.

Furthermore, we determined the second Hankel determinant and established relevant radius results for the new class. These results provide deeper insight into the analytic structure of the subclass and offer a unified framework that connects the operator-based approach with classical analytic function theory.

The obtained results have potential applications in modeling stability regions and dynamic behaviors of systems arising in applied sciences such as fluid dynamics, electromagnetism, and health modeling. The radius estimates, in particular, can be interpreted as describing regions of guaranteed functional stability in such models.

Future investigations may consider extending this study to higher-order coefficient estimates, Hankel determinants, and Fekete–Szegö inequalities for related subclasses generated by other orthogonal polynomial operators or fractional differential operators.

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